Theory and Practice of Science and Technology
2025, VOL. 6, NO. 5, 80-86
DOI: 10.47297/taposatWSP2633-456917.20250605

Simplified Feedforward ANC Algorithm Based on Adaptive Step Size

Jiawei Cai

Department of Physics China Jiliang University, Hangzhou 310018, China

ABSTRACT
In feedforward active noise control (ANC) systems, fixed step-size algorithms often struggle to achieve both rapid convergence and low steady-state error. To address this limitation, this study proposes a simplified feedforward ANC algorithm based on an adaptive step-size strategy, derived from the combined characteristics of logarithmic and modified Sigmoid functions. Simulation experiments using white noise as the input source demonstrate that the proposed algorithm not only accelerates convergence but also significantly reduces steady-state error, confirming its feasibility and practical potential.

KEYWORDS
active noise control; adaptive step size; feedforward system

1 Introduction

ANC systems are categorized into feedforward active noise control systems and feedback active noise control systems. In feedforward ANC systems, the reference signal is directly obtained from the noise source via a reference microphone. In contrast, feedback ANC systems cannot directly acquire the primary noise as a reference signal; instead, the reference signal is usually generated or estimated within the system itself [1].

Traditional feedforward ANC algorithms use a fixed step size, which significantly affects the stability and convergence speed of the algorithm. A larger step size can accelerate convergence but may result in a larger steady-state error, whereas a smaller step size can reduce the steady-state error but slows down the convergence. Therefore, fixed-step-size ANC algorithms cannot simultaneously achieve both fast convergence and low steady-state error. To address this issue, many researchers have extensively studied variable step-size ANC algorithms. Liu Ningning et al. proposed an iterative variable step-size ANC algorithm based on the arctangent function, applying it to suppress engine-order noise at the driver's ear in a vehicle[2]. The study demonstrated that this algorithm effectively reduces in-vehicle engine-order noise. Li Tao et al. introduced a variable step-size ANC algorithm based on the Sigmoid function, which achieves good convergence speed and low steady-state error[3]. Using low-frequency train noise as the input, simulation experiments verified its effective noise reduction performance. Ru Guobao et al. proposed a new variable step-size ANC algorithm based on the logarithmic function[4]. Zhang Shuai et al. combined normalization with the arctangent function to develop an adaptive step-size ANC algorithm[5]. These algorithms, to some extent, address the issue of traditional ANC algorithms being unable to ensure both fast convergence and low steady-state error. However, existing variable step-size algorithms based on a single nonlinear function still have limitations, making it difficult to simultaneously achieve rapid convergence and low steady-state error. By analyzing the characteristics of the logarithmic-function-based adaptive step-size algorithm and the improved Sigmoid-function-based adaptive step-size algorithm, it is observed that the logarithmic function enables faster convergence, while the improved Sigmoid function maintains a smaller steady-state error. Therefore, this paper proposes a novel segmented adaptive step-size function combining the logarithmic and improved Sigmoid functions and applies it to a feedforward ANC algorithm, resulting in a Simplified Feedforward Active Noise Control algorithm based on Adaptive Step.

2 The proposed simplified feedforward ANC algorithm based on adaptive step size

2.1 Algorithm principle

A fixed step-size parameter cannot simultaneously satisfy the requirements for both convergence speed and algorithmic stability. When the error signal is large, a larger step size should be selected to accelerate convergence; conversely, when the error signal is small, a smaller step size should be used to ensure better steady-state performance. To address this issue, this paper introduces a nonlinear function to establish a dynamic adjustment relationship between the error signal $e(n)$ and the step size $\mu(n)$. In this relationship is described using a logarithmic function, which can be expressed as:

$$\mu(n) = \alpha_1 \log(\beta_1 |e(n)|^{c_1}) \tag{2-1}$$

In Equation (2-1), the parameters α_1、β_1、and c_1 determine the range of the step-size function. Using the method of controlling variables—i.e., fixing two parameters while varying one—allows us to analyze the influence of each parameter on the step-size behavior.

(1) When $\beta_1 = 5000$ and $c_1 = 2$ are fixed, the the impact of α_1 on the exponential step-size function defined in Equation (2-1) is analyzed, as illustrated in Figure 1. When $\alpha_1 = 0.8$, the maximum value of the step size $\mu(n)$ is approximately 0.74; when $\alpha_1 = 1.0$, the maximum value increases to approximately 0.92; and when $\alpha_1 = 1.2$, it further increases to about 1.10. These results indicate that the step size $\mu(n)$ increases with increasing α_1.

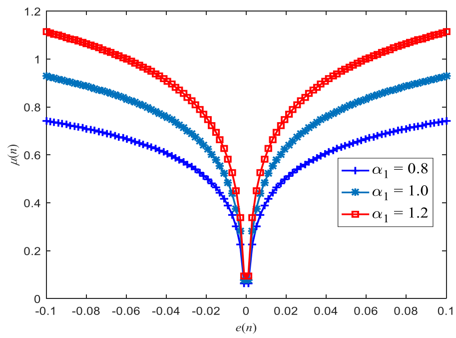

Figure 1 Effect of parameter α_1 variation on the $e(n)$-$\mu(n)$ curve of the algorithm

(2) When $\alpha_1 = 0.5$ and $c_1 = 2$ are fixed, the the impact of c_1 on the exponential step-size function defined in Equation (2-1) is analyzed, as illustrated in Figure 2. When $\beta_1 = 1.5$, the maximum value of the step size $\mu(n)$ is approximately 0.92; when $\beta_1 = 1.75$, the maximum value increases to approximately 1.07; and when $\beta_1 = 2.0$, it further increases to about 1.18. These results indicate that the step size $\mu(n)$ increases with increasing β_1.

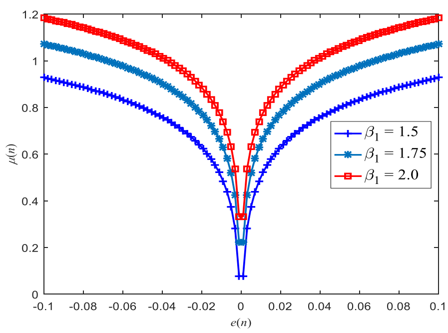

Figure 2 Effect of parameter β_1 variation on the $e(n)$-$\mu(n)$ curve of the algorithm

(3) When $\alpha_1 = 0.12$ and $\beta_1 = 3000$ are fixed, the the impact of c_1 on the exponential step-size function defined in Equation (2-1) is analyzed, as illustrated in Figure 3. When $c_1 = 2$, the maximum value of the step size $\mu(n)$ is approximately 0.90; when $c_1 = 3$, the maximum value increases to approximately 0.75; and when $c_1 = 4$, it further increases to about 0.72. These results indicate that the step size $\mu(n)$ increases with increasing c_1.

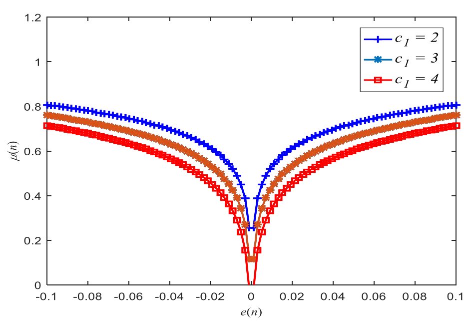

Figure 3 Effect of parameter c_1 variation on the $e(n)$-$\mu(n)$ curve of the algorithm

From the above analysis, it can be seen that the logarithmic step-size function in Equation (2-1) changes smoothly at the upper end of the curve, allowing the algorithm to maintain a relatively large step size during the early stages of iteration, thereby achieving a faster convergence rate. However, the function becomes steeper at the lower end of the curve, resulting in a relatively large and rapidly changing step size in the later stages of iteration, which leads to increased steady-state error. Therefore, the main advantage of the logarithmic adaptive step-size algorithm lies in its fast convergence, while its disadvantage is the relatively high steady-state error as the algorithm progresses.

In the improved Sigmoid step-size function[6], the adjustment relationship between the error signal $e(n)$ and the step size $\mu(n)$ is given by:

$$\mu(n)=\alpha_2(1-\exp(-\beta_2\left|e(n)\right|^{c_2})) \tag{2-2}$$

In Equation (2-2), the parameters α_2, β_2 and c_2 determine the range of values for the step-size function. To analyze the influence of each parameter on the function, a controlled variable method is employed, in which two parameters are held constant while the third is varied..

When the parameters $\beta_2 = 5$ and $c_2 = 2$ are fixed, the influence of different values of α_2 on the improved Sigmoid step-size function in Equation (2-2) is analyzed, as shown in Figure 4. When $\alpha_2 = 10$, the maximum value of $\mu(n)$ is approximately 0.48; when $\alpha_2 = 15$, the maximum value of is approximately of $\mu(n)$ 0.73; and when $\alpha_2 = 20$, the maximum value of is approximately 0.97. These results indicate that the step size increases with increasing α_2.

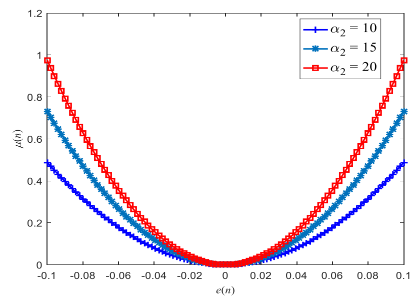

Figure 4 Effect of parameter α_2 variation on the $e(n)$-$\mu(n)$ curve of the algorithm

When the parameters $\alpha_2 = 10$ and $c_2 = 2$ are fixed, the influence of different values of β_2 on the improved Sigmoid step-size function in Equation (2-2) is analyzed, as shown in Figure 5. When $\beta_2 = 3$, the maximum value of $\mu(n)$ is approximately 0.30; when $\beta_2 = 5$, the maximum value of is approximately of $\mu(n)$ 0.49 and when $\beta_2 = 10$, the maximum value of is approximately 0.95. These results indicate that the step size increases with increasing β_2.

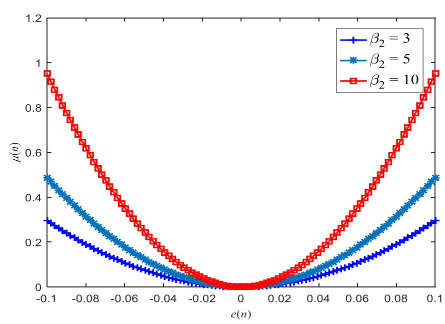

Figure 5 Effect of parameter β_2. variation on the $e(n)$-$\mu(n)$ curve of the algorithm

When the parameters $\alpha_2 = 20$ and $\beta_2 = 5$ are fixed, the influence of different values of c_2 on the improved Sigmoid step-size function in Equation (2-2) is analyzed, as shown in Figure 6. When $c_2 = 2$, the maximum value of $\mu(n)$ is approximately 0.97; when $c_2 = 3$, the maximum value of is approximately of $\mu(n)$ 0.31 and when $c_2 = 4$, the maximum value of is approximately 0.10. These results indicate that the step size $\mu(n)$ decreases with increasing c_2.

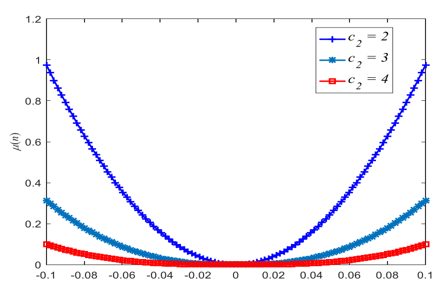

Figure 6 Effect of parameter c_2 variation on the $e(n)$-$\mu(n)$ curve of the algorithm

The improved Sigmoid step-size function in Equation (2-2) has a relatively sharp curve at the top. In the early stages of algorithm iteration, the step size decreases rapidly, which may cause it to approach zero prematurely, thereby slowing down the convergence. In contrast, during the later stages of iteration, the bottom of the curve changes gradually and smoothly, allowing the algorithm to use smaller step sizes to reduce steady-state error. Therefore, the advantage of the improved Sigmoid step-size function lies in achieving a smaller step size in the later stages, which helps reduce steady-state error, while the drawback is its relatively slow convergence.

Based on the above analysis, although using a single nonlinear function to adjust the relationship between the error signal and the step size can, to some extent, balance steady-state error and convergence speed, it still has limitations. To address this issue, this paper proposes an adaptive step-size method based on a piecewise function, which integrates the advantages of both the logarithmic and improved Sigmoid step-size algorithms. In the early stage of iteration, the logarithmic function is used to regulate the step size for faster convergence, while in the later stage, the improved Sigmoid function is used to reduce steady-state error. Thus, a piecewise adaptive step-size adjustment formula is derived and can be expressed as:

$$\mu(n) = \begin{cases} \alpha_3 \log(\beta_3 |e(n)|^{C_3}) & |e(n)| > \tau \\ \alpha_4 (1 - \exp(-\beta_4 |e(n)|^{c_4})) & |e(n)| > \tau \end{cases} \tag{2-3}$$

In Equation (2-3), the parameters α_3, α_4, β_3, β_4, c_3, and c_4 are used to control the shape of the step-size function curve, and the performance of the algorithm is significantly influenced by these parameters. During simulation, it is necessary to adjust these parameters repeatedly—using methods such as the control variable approach or trial-and-error—based on the specific application scenario, to ensure optimal algorithm performance. The threshold τ determines when to switch between different step-size adjustment formulas in the piecewise function. If τ is set too large or too small, the effectiveness of the piecewise adaptive step-size strategy will be compromised. Typically, τ is chosen to correspond to the magnitude of the error signal when the algorithm is nearing convergence, and its value usually ranges from 10% to 30% of the maximum error signal. After extensive experimentation, this study sets τ to 10% of the maximum error signal.

2.2 Algorithm derivation

The block diagram of the simplified feedforward Active Noise Control (ANC) algorithm based on adaptive step size is shown in Figure 7. In the diagram, $x(n)$ represents the noise source signal, and $W(z)$ denotes the adaptive filter. The noise signal $x(n)$ is processed by the filter $W(z)$ to produce the output signal $y(n)$, which then passes through the secondary path to yield $y'(n)$. Simultaneously, $x(n)$ also passes through the primary path to generate the desired signal $d(n)$. The error signal $e(n)$ is defined as the difference between the desired signal $d(n)$ and the secondary path output $y'(n)$. $x_f(n)$ is the filtered reference signal.

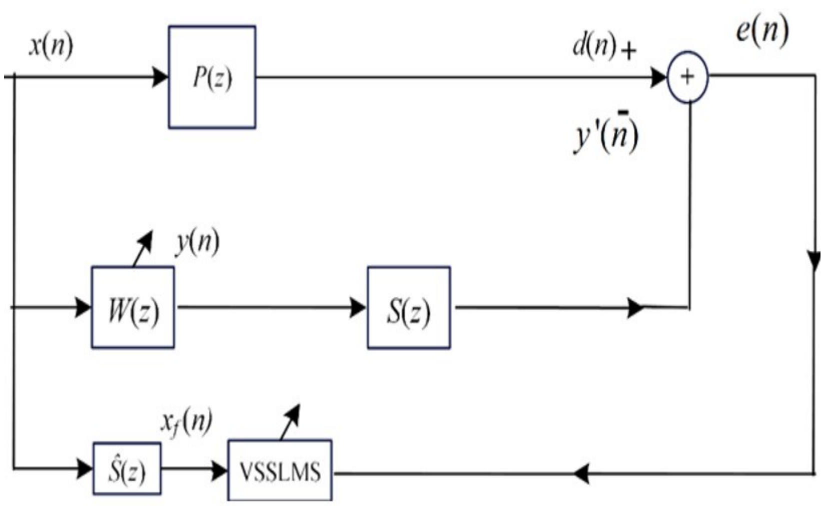

Figure 7 Block diagram of the simplified feedforward ANC algorithm based on adaptive step size

The noise source $x(n)$ is passed through a filter $W(z)$ of length L, resulting in the output signal $y(n)$ given by:

$$y(n) = \boldsymbol{w}^T(n)\boldsymbol{x}_L(n) \tag{2-4}$$

In this equation, $\boldsymbol{w}(n)$ denotes the coefficient vector of the adaptive filter, defined as $\boldsymbol{w}(n) = [w_0(n), w_1(n), \cdots, w_{L-1}(n)]^T$, and $x_L(n)$ is the input signal vector of length L, given by $\boldsymbol{x}_L(n) = [x(n), x(n-1), \cdots, x(n-L+1)]^T$.

After passing through the secondary path, the output signal $y(n)$ becomes $y'(n)$, received at the error sensor, expressed as:

$$\boldsymbol{y'(n)} = \boldsymbol{s(n)}^T \boldsymbol{y_M(n)} \tag{2-5}$$

Here, $\boldsymbol{s}(n)$ is the impulse response of the secondary path with length M, and $y_M(n)$ is the signal vector of length M, defined as $\boldsymbol{y}_M(n) = [y(n), y(n-1), \cdots, y(n-M+1)]^T$.

The error signal $e(n)$ is obtained as the difference between the desired signal $d(n)$ and the secondary path output $y'(n)$, formulated as:

$$e(n) = d(n) - y'(n) \tag{2-6}$$

The proposed adaptive step size control algorithm based on a piecewise function is defined by:

$$\mu(n) = \begin{cases} \alpha_3 \log(\beta_3 |e(n)|^{C_3}) & |e(n)| > \tau \\ \alpha_4 (1 - \exp(-\beta_4 |e(n)|^{c_4})) & |e(n)| > \tau \end{cases} \tag{2-7}$$

In Equation (2-7), α_3, α_4, β_3, β_4, c_3, c_4 are parameters that control the shape of the function. The threshold τ determines the switching point between the two step-size functions and thus affects when different update strategies are used during the iteration. A poorly chosen τ may impair the effectiveness of the piecewise approach.

Using the gradient descent method, the adaptive filter coefficient update formula is derived as:

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \mu(n) \boldsymbol{x}_f(n) e(n) \tag{2-8}$$

Here, $x_f(n)$ is the filtered reference signal vector of length M, defined as: $\boldsymbol{x}_f(n) = [x_f(n), x_f(n-1), \cdots, x_f(n-M+1)]^T$.

The derivations from Equation (2-4) to Equation (2-8) together form a Simplified Feedforward Active Noise Control algorithm based on Adaptive Step size (SFASANC).

3 Experimental Results

The system is configured with 3×10^5 sampling points and a sampling frequency of 2400 Hz. The primary noise source is white Gaussian noise with a mean of zero and a variance of one. The logarithmic adaptive step size algorithm (LASANC) and the improved sigmoid adaptive step size algorithm (ISASANC) are applied to a feedforward active noise control (ANC) system and compared with the proposed SFASANC algorithm in terms of noise reduction performance and convergence characteristics.

Under the premise of ensuring the stability of all three algorithms, extensive experiments were conducted to determine the optimal parameters for the step size ?(?) in each algorithm. The parameter settings for the LASANC and ISASANC algorithms are listed in Table 1, while those for the SFASANC algorithm are shown in Table 2.

Table 1 Parameter settings of the LASANC and ISASANC algorithms

	LASANC			ISASANC		
parameter	α_1	β_1	c_1	α_2	β_2	c_2
value	0.3	10	0.4	8.5	0.5	2

Table 2 Parameter settings of the LASANC and ISASANC algorithms

	SFASANC						
parameter	α_3	β_3	c_3	α_4	β_4	c_4	τ
value	0.35	10	0.25	9	0.3	2	0.01

To ensure a fair comparison among the three algorithms, the maximum value of the step-size function $\mu(n)$, corresponding to the maximum value of $|e(n)|$, is set to be the same for all methods. In the early stages of the algorithm, the piecewise adaptive step-size function based on the logarithmic and improved Sigmoid functions can yield larger step sizes than the logarithmic-based method, resulting in faster convergence. In the later stable phase, it can provide smaller step sizes than the improved Sigmoid-based method, thereby achieving a lower steady-state error, as shown in Figure 8.

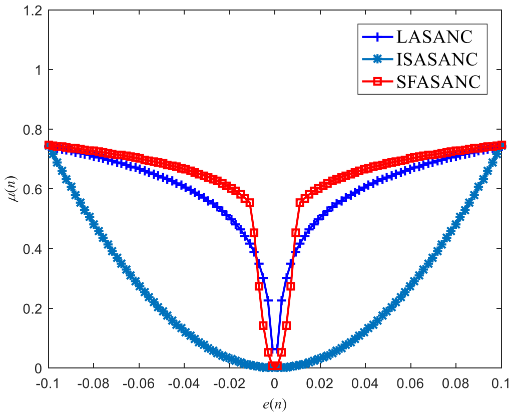

Figure 8 Three step-size adjustment formulas

The Mean Squared Error (MSE) is defined as follows:

$$\boldsymbol{MSE} = 10 * \log\left(\frac{1}{\boldsymbol{n}}\sum_{\boldsymbol{i}=1}^{\boldsymbol{n}} \boldsymbol{e}_{\boldsymbol{i}}^{2}\right) \tag{3-1}$$

where n is the number of system sampling points, and e_i denotes the error signal at the i-th iteration.

A comparison of the MSE curves for the three algorithms is shown in Figure 9. It can be observed that the proposed SFASANC algorithm achieves the fastest convergence while maintaining the lowest steady-state error. The convergence speeds of LASANC and ISASANC are comparatively slower. Due to the nonlinear relationship introduced by the logarithmic – improved sigmoid piecewise step size function between $\mu(n)$ and $e(n)$, the SFASANC algorithm demonstrates faster convergence than ISASANC, and it attains a lower steady-state error than LASANC.

Therefore, the SFASANC algorithm effectively combines the advantages of both LASANC and ISASANC. At steady state, the MSE achieved by SFASANC is approximately – 28 dB, compared to – 26 dB for ISASANC and – 24 dB for LASANC, indicating superior noise reduction performance of the proposed algorithm.

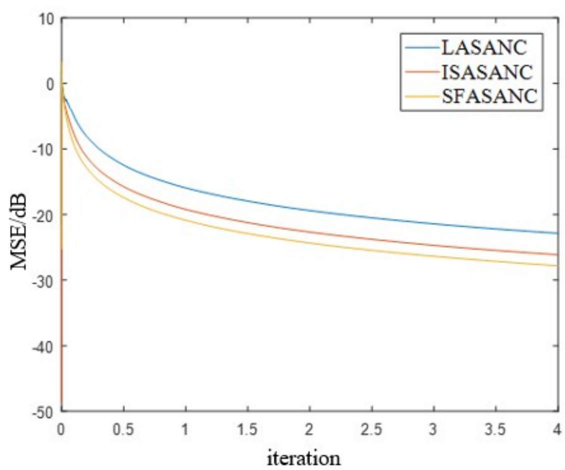

Figure 9 The mean squared error curves of the three algorithms

4 Summary

An analysis of the convergence characteristics of fixed step-size algorithms reveals that the step size significantly affects both the convergence speed and the steady-state error. Specifically, a smaller step size can reduce the steady-state error but slows down convergence, whereas a larger step size accelerates convergence but often leads to higher steady-state error. Moreover, a single adaptive step-size function is typically insufficient to optimize both convergence rate and steady-state accuracy simultaneously.

To address this issue, this paper proposes a Simplified Feedforward Active Noise Control algorithm based on Adaptive Step Size (SFASANC). By employing a piecewise-defined adaptive step-size function, the algorithm effectively balances the trade-off between fast convergence and low steady-state error.

Simulations were conducted using zero-mean, unit-variance white noise as the input signal to compare the convergence and noise reduction performance of three adaptive step-size ANC algorithms based on the logarithmic function, the improved sigmoid function, and the proposed logarithmic – improved sigmoid piecewise function. The results demonstrate that the proposed SFASANC algorithm achieves faster convergence and lower steady-state error compared to the other two approaches. In steady state, the SFASANC algorithm achieves a minimum mean squared error (MSE) of approximately −28 dB, indicating superior noise attenuation performance.

References

[1] XIE X, WANG Y. An Improved FxLMS Method Based on Complete Ensemble Empirical Mode Decomposition with Adaptive Noise; proceedings of the 2024 6th International Conference on Natural Language Processing (ICNLP), F, 2024 [C]. IEEE.

[2] Liu Ningning, Sun Yuedong, Wang Yansong, et al. Experimental study on active control system for automobile engine noise [J]. Automotive Engineering, 2022, 44(3): 442–448.

[3] Li Tao, He Yuyao, Wang Ning, et al. Feedforward-reference-based variable-convergence train noise active control algorithm [J]. Electric Drive for Locomotives, 2024, (1): 25–30.

[4] Ru Guobao, Huang Yan, Guo Yingjie, et al. New variable step size LMS algorithm based on logarithmic function [J]. Journal of Wuhan University: Natural Science Edition, 2015, (3): 295–298.

[5] Zhang Shuai, Wang Yansong, Zhang Xinguang, et al. VS-MFxLMS algorithm and its application in in-vehicle noise active control [J]. Noise and Vibration Control, 2019, 39(2): 64–69.

[6] Li Lijun, Wang Huakui. An improved variable step size LMS adaptive filtering algorithm [J]. Modern Electronic Technology, 2006, 29(7): 7–9.